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DETERMINATION OF FAILURE CHARACTERISTICS OF MATERIALS AND STRUCTURES

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DETERMINATION OF FAILURE CHARACTERISTICS OF MATERIALS AND STRUCTURES

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ABSTRACT

A research program has been pursued with the objective of examining failure characteristics of materials and structures analytically and experimentally. Several aspects of the problem have been studied, which are discussed under different sections.

The expression for the nonlinear energy toughness was rederived for a generalized instability condition and new expressions were derived for biaxial loading situations. The geometry dependence of the nonlinear energy toughness was studied experimentally using center-cracked thin sheet specimens and thicker compact tension specimens of several alloys and compared with other nonlinear toughness parameters. The nonlinear energy toughness can be used as a useful failure criterion under certain geometrical variations in semi brittle materials. Its geometry dependence is similar to that of J_{Ic}. Experimental and finite element studies have shown that the plastic energy dissipation is a linear function of the stable crack growth, but the slope of the curve has been found to change considerably with changes in crack length-to-specimen width ratio. Singular solutions of cracked bodies predict no biaxial effect when one load axis is parallel to the crack plane. However, it has been shown analytically that including the second term reveals changes in stresses and strains in the vicinity of the crack tip due to biaxiality. Finite element studies show larger dependence

of stresses and strains on biaxiality on ductile materials. In experimental studies the biaxial effects have been found to be small. Several parameters affecting buckling of spherical shells have been studied. Pulse duration has a significant effect on buckling under dynamic loading. Plastic deformation has a significant effect in buckling only when the imperfection is small, and creep plays an important role in long term loading.

A finite element program incorporating incremental plasticity constitutive relations, arbitrary hardening law and multilinear stress-strain curve has been developed to study plastic deformation at crack tip. Under the ongoing research program, these results are being compared with experimental determinations. This work will be extended to the stable crack growth region. A fracture parameter with predictive capability in envisioned on the major outcome of this research.

TABLE OF CONTENTS

I. INTRODUCTION

The research program, Determination of Failure Characteristics of Materials and Structures, was initiated for the purpose of broadening and strengthening the scientific base for the prediction and prevention of failure in structures of interest to the Navy. A coordinated approach involving analytical, finite element and experimental studies was followed to insure realistic assumptions and verification from actual behavior of engineering materials.

Linear fracture mechanics concepts have been well accepted by the scientific community and fracture toughness (K_{Ic}) is being used by the industries as a criterion for crack growth resistance in the design of structures. The search for high fracture toughness led to the development of the modern high performance alloys, which are widely used by the Department of Defense. However, in high toughness alloys the analysis of crack tip stresses and development of a fracture criterion become difficult due to considerable crack-tip plasticity and stable crack growth. When this research program was initiated the understanding of the nonlinear effect in fracture was limited. Because of the complex nature of the problem, no suitable criterion for semibrittle fracture existed. Furthermore, the application of any fracture criterion to structures had to take into account the modification of stresses produced by the geometry and multiaxial loading.

Hence in this program considerable attention was given to nonlinear, biaxial and geometrical effects in fracture. The research program covered a wide range of topics dealing with fracture of engineering materials. The research effort can be broadly divided into the following related areas.

1. The development of the nonlinear energy fracture toughness ($\tilde{G}_c$) and assessment of several nonlinear toughness parameters as fracture criteria.
2. Plastic energy dissipation during stable crack growth.
3. Effect of biaxial loading on fracture.
4. Buckling of spherical caps.
5. Plastic deformation in three-dimensional fracture specimens.

The extensive research in these areas made important contributions to the understanding of failure of structural materials used by the Navy. It also resulted in several presentations in conferences and publications in scientific journals, copies of which are furnished in Appendix A. In the following sections, only the important accomplishments will be highlighted. Details are given in the appropriate sections of Appendix A, cited at the end of each section.

II. THE DEVELOPMENT OF $\tilde{G}_c$ AND ASSESSMENT OF NONLINEAR FRACTURE TOUGHNESS CRITERIA

For fracture under circumstances where crack-tip plastic deformation may be too extensive to be ignored or treated as a minor correction, and where fast fracture may be preceded by substantial subcritical crack growth, defining and evaluating a fracture toughness criterion is difficult. Hence, several measures of fracture toughness have been proposed of which the parameters based on the R-curve (G_R), the crack opening displacement (G_δ), the J-integral (J_c) and the nonlinear energy method ($\tilde{G}_c$) are well known.

The nonlinear energy method was initially suggested by Liebowitz and Eftis [1,2] as a correction to linear fracture toughness to account for the nonlinearity. It was later developed as an independent criterion based on the energy balance during crack growth [3,4].

When a fracture test is performed slowly and adiabatically the energy balance equation can be written as

$$\frac{\partial W}{\partial c} = \frac{\partial U'}{\partial c} + \frac{\partial U''}{\partial c} + \frac{\partial \Gamma}{\partial c} \tag{1}$$

in which W = the external work,

U' = the elastic energy,

U'' = the plastic energy,

Γ = the fracture surface energy,

and c = the crack length.

Based on this definition the nonlinear energy toughness ($\tilde{G}_c$) is defined as

$$\tilde{G}_c = \frac{\partial W}{\partial c} - \left(\frac{\partial U'}{\partial c} + \frac{\partial U''}{\partial c}\right) = \frac{\partial \Gamma}{\partial c} \ . \tag{2}$$

From this general definition, an expression for $\tilde{G}_c$ has been developed. The load displacement (F-v) curve obtained from a fracture toughness test of a ductile material can be represented by a Ramberg-Osgood type relation in the form

$$v = \frac{F}{M} + k\left(\frac{F}{M}\right)^n \tag{3}$$

in which M is the initial specimen stiffness and k and n are constants. From Equations (2) and (3) an expression for $\tilde{G}_c$ has been obtained as

$$\tilde{G}_c = \left[1 + \frac{2nk}{n+1}\left(\frac{F}{M}\right)^{n-1}\right] G_c = \tilde{C}\ G_c \tag{4}$$

This expression was derived assuming fixed grip [1] as well as fixed-load [4] condition at instability.

One task undertaken in this area of research was further development of the nonlinear energy method analytically and evaluation of the applicability of this criterion to real materials. The results were also compared with other toughness criteria.

Although expression (4) was derived assuming fixed grip on fixed load condition at instability, it has been observed that neither of these conditions exists during fracture.

Hence, this expression has been rederived assuming arbitrary changes in load and displacement at instability [5].

The usefulness of a fracture toughness criterion in engineering applications can be estimated by its dependence on changes in geometry and size. Hence, $\tilde{G}_c$ and several other toughness parameters were examined and compared as fracture criteria by testing several alloys in different specimen configurations and sizes [6,10]. In specimens with thickness lower than that required by ASTM Standard E399, considerable stable crack growth occurred before final fracture. For these specimens toughness values were determined at two critical points, (a) the point of initiation of stable crack growth and (b) the peak load. Center-cracked specimens were prepared from thin sheet materials and compact tension specimens were prepared from thicker plates.

The expression for nonlinear energy toughness (4) was derived assuming the existence of nonlinear deformation, but no subcritical crack growth. As thin sheet specimens generally exhibit appreciable subcritical crack growth, expression (4) would appear to need modification. To estimate the effect of subcritical crack growth, four plausible methods of accounting for this effect were considered [8]. From the experimental results it was concluded that using the original crack size and critical load in expression (4) represented the most simple and straightforward procedure and also provided results in good agreement with the other three methods.

Aluminum alloys 2024-T3 and 7075-T6 were tested in sheet form as center-cracked specimens of thickness 0.063 in. [7,8]. The length, the width and the crack length-to-specimen width ratio were varied in these specimens. Since the only other criterion suggested for center-cracked specimens involving considerable stable crack growth was the R-curve toughness (G_R), $\tilde{G}_c$ was compared with G_R at peak load. At the point of crack growth initiation nonlinear energy toughness ($\tilde{G}_o$) was compared with the linear toughness ($\overline{G}_o$). The peak load values $\tilde{G}_c$ and G_R increased with decreasing length and crack length-to-width ratio and increasing width. $\tilde{G}_c$ and G_R generally exhibited the same dependence on specimen geometry, although $\tilde{G}_c$ was consistently 5-10% higher than G_R for 7075-T6 and 10-20% higher for 2024-T3. At the point of crack growth initiation, $\tilde{G}_o$ and $\overline{G}_o$ were essentially independent of specimen geometry, within the ranges tested. However, these toughness values were often less than one-half of the corresponding values at peak load.

In compact tension specimen tests the effect of variation in specimen thickness and width (W) on toughness parameters $\tilde{G}_{Ic}$, J_{Ic}, G_δ and $\overline{G}_{Ic}$ was determined. They were determined at the onset of stable crack growth and at peak load in this series also. The thicknesses tested included those above and below that required by ASTM E399 for plane strain fracture. The materials tested were aluminum alloys 7075-T651, 2124-T851, 2048-T351 and 2048-T851, titanium alloy

Ti-6Aℓ-4V and 4340 steel. Several interesting conclusions were reached from these tests [6,7,9,10]. The fracture toughness values determined at the point of crack growth initiation, $\overline{G}_{Io}$, $\tilde{G}_{Io}$ and J_{Io} are independent of thickness from above to well below the minimum value required by ASTM E399. The nonlinear material response is very limited prior to the initiation of stable crack growth even for somewhat ductile materials. The toughness values $\overline{G}_{Io}$, $\tilde{G}_{Io}$ and J_{Io} are essentially the same for all tests of each material where thickness independence is observed, and are comparable with the fracture toughness values determined by E399, even though the thickness requirement is not met. The fracture toughness parameters determined at peak load exhibit strong thickness dependency. (The G_{δ} values exhibited considerable scatter and inconclusive results, probably due to the limitations in the method of determining crack opening displacement.) At any given thickness, the fracture toughness values increase with increasing specimen size (w). The $\tilde{G}_{Ic}$ and J_{Ic} toughness values behave in a similar fashion, as expected from the similarity of their definitions.

An important assumption made in the derivation of expression (4) for $\tilde{G}_c$ the load-displacement curve can be defined by the relationship (3) and n and k are constant independent of small changes in crack length. Hence, an independent assessment of the nonlinear energy method was made using a finite element analysis from the general definition (2). Instead of using

Equation (3), the load-displacement curve was developed from the stress-strain curve obtained in smooth specimen tensile test [11,12]. Since part of the specimen experienced unloading the problem was formulated based upon incremental plasticity theory. Where subcritical crack growth was analyzed, experimental load-crack growth results were also utilized. The conclusions arrived at from the four methods used in experimental evaluation of subcritical crack growth [8] were also examined independently using finite element analysis. $\tilde{G}_c$ values obtained from finite element at the onset of subcritical crack growth as well as at the peak load were in good agreement with the experimental results [12].

Note: Details of the study are furnished in Appendix A: A-1-A-5, A-19, A-20 and A-26.

III. PLASTIC ENERGY DISSIPATION DURING STABLE CRACK GROWTH

Ductile materials exhibit considerable crack-tip plasticity and stable crack growth prior to fracture [8,10]. The extent of stable growth increases as the thickness decreases from that required for plane strain fracture. It has been observed that plastic deformation at the initiation of stable crack growth is small compared to the total plastic strain before unstable fracture in sheet specimens. Hence, the changes in the pattern of plastic deformation has an important role in the total amount of stable crack growth and final fracture. A better understanding of the stable crack growth processes has been attained through the experimental and finite element studies performed under this contract [11-17].

Experimental investigation of plastic energy dissipation before and during stable crack growth was performed on center-cracked thin sheet specimens of 2024-T3 aluminum alloy and its relationship with fracture toughness parameters was studied [13]. The dissipated plastic energy was calculated by subtracting the elastic energy from the total energy. Tests were performed in both load control and displacement control modes. The effect of changes in the geometry and size of the specimen was studied by varying width and crack length-to-width ratio of the specimens. For all crack lengths and widths an approximately linear relationship between plastic energy

dissipation and stable crack growth was observed. Although considerable scatter in plastic energy dissipation rate (dP/da) was observed at a given crack length, and width, corresponding variation in the load-displacement curve and fracture load was negligible. The dissipation rate (dP/da) was found to be highly sensitive to crack length-to-width ratio (2a/w); it sharply dropped with increase in 2a/w. Hence, its predictive capability as a geometry-independent parameter is not significant. A strong correlation between dP/da and fracture toughness parameters $\tilde{G}_c$ or G_R was not observed.

The linear relationship between plastic energy dissipation and stable crack growth provided a convenient tool for numerical modeling. A computer program for finite element analysis incorporating incremental plasticity theory with an isotropic hardening law was developed [11,12,14-16]. The crack was grown in small increments by releasing the crack tip nodal force and allowing the redistribution of the plastic zone. From the experimental load-crack growth and load-displacement curves as input, the plastic energy dissipation rate was obtained, which was found to be constant during slow crack growth as seen experimentally. Conversely from a knowledge of the point of crack growth initiation and the linear relationship between plastic energy dissipation and crack growth, it was possible to obtain the load-crack growth and load-displacement curves. This reciprocity demonstrated the ability of the program to model slow growth accurately, given a suit-

able growth constitutive relation. Further numerical studies showed that the material nonlinearity could be modeled equally well with either a Ramberg-Osgood or a multilinear stress-strain model. A unique feature of this analysis is that, given the slope of the plastic energy versus crack length curve, the finite element analysis is able to predict the load at instability by calculating the load at which the tangent to the load-displacement curve becomes zero. Thus, the final crack length and peak load can be predicted by the analysis. The computer code was modified to account for large deformation, which has considerable influence on the solution for problems involving large plastic zones [17]. This modification allowed a wide range of problems to be analyzed accurately. As observed in small strain analysis, a linear relationship between plastic energy dissipation and stable crack growth was obtained, but the slope obtained from the two analyses were significantly different. Although the plastic energy dissipation rate was constant, the plastic zone size was found to increase with stable crack growth, confirming the observation made in an earlier experimental study [18].

Along with the evaluation of plastic energy dissipation during slow crack growth, the finite element analysis was also used to determine various other fracture parameters [16]. Experimental load-crack growth curves from 2024-T3 aluminum alloy specimens were taken as input to the computer program.

The nonlinear energy release rate, J-integral, crack opening angle, crack-tip strain and stress, remote displacements and local strain energy density were calculated as a function of crack growth. The nonlinear energy release rate increases monotonically, however the functional form and instability values are geometry dependent. The J-integral is not only geometry dependent but also path dependent. The crack opening angle and local strains do not vary consistently. Both crack opening stress and strain energy density remain reasonably constant during slow crack growth. Due to the inability to measure these parameters accurately in experiments, their usefulness as possible crack growth characterizations is limited.

The experimental and theoretical studies summarized demonstrate the limitations and inadequacies of the existing fracture parameters to predict crack growth and failure characteristics of ductile fracture problems. While these studies have provided valuable insight into the process of ductile fracture, additional research work is needed to provide a truly predictive theory for slow crack growth problems.

Note: Details of the study are furnished in Appendix A: A-3, A-4, A-11, A-23-A-25 and A-27.

IV. THE EFFECT OF LOAD BIAXIALITY ON FRACTURE

For all geometries and outer boundary loading conditions, it is more or less accepted in fracture mechanics that the elastic stress and displacement very near the tip of a plane line crack can be approximated with sufficient accuracy by a one parameter representation in terms of the stress intensity factors K_I and K_{II}. In an earlier work by Eftis, Subramonian and Liebowitz [19] it was shown that this assumption is not generally applicable. The error can be attributed to the practice of arbitrarily omitting the second term which is independent of the distance from the crack tip. The inclusion of this term changes the calculation of stresses under biaxial loading conditions. The stress distribution in biaxial loading is affected more significantly in ductile materials than in brittle materials due to the changes induced by crack-tip plastic deformation. Under this research program the importance of these aspects in biaxial loading has been examined analytically, experimentally and using finite element techniques [11,20-29].

Analytically the effect of including the second term in the expressions for stress and displacement was examined in an infinitely wide center-cracked specimen loaded biaxially with the crack parallel to one of the load axes. The effect of the load applied parallel to the plane of the crack appears entirely in the second term of the series expansion [20,21].

Thus, in the calculation of stress, displacement and related quantities of interest in the crack border region by means of the standard expressions, no biaxial load effects will appear, leading thereby to the erroneous impression that load applied parallel to the plane of the crack can have no influence with regard to the fracture problem. On including the second term, significant biaxial load effect is observed on crack edge displacement, local maximum shear stress, pattern of maximum shear isostats, angle of initial crack extension and local strain energy and strain energy rate. On the other hand, the elastic K and J-integral show no sensitivity to the presence of the load parallel to the crack. In sheets with slanted crack, initial angles of crack extension and path of crack extension for crack orientation were examined based on maximum opening stress and minimum strain energy density [26]. The initial angle of crack extension obtained is a function of r, the distance from the crack tip at which these parameters are evaluated. When r tends to zero the angle is the same as that obtained from the singular solution. Although these results are based on elastic analysis, they are useful in outlining a procedure which could be used when considering the elastic-plastic problem of crack extension.

The compliance analysis used for the development of the nonlinear energy method in uniaxial loading was generalized for mixed nonlinear energy toughness determination [28]. For a specimen with an arbitrary crack under biaxial loads (F_x and

F_y), the energy release rate for the linear portion of the compliance curve can be written as

$$\tilde{G}_c = G_x + G_y + G_{xy}$$

$$= \frac{1}{4B} [F_x^2 \frac{\partial G_x}{\partial a} + F_y^2 \frac{\partial C_y}{\partial a} + 2F_xF_y \frac{\partial C_{xy}}{\partial a}] \quad (5)$$

where C_x, C_y and C_{xy} are the linear compliance coeffieients. Assuming that the linear and nonlinear displacement are functionally related and separable, the nonlinear energy release rate can be written in terms of the linear decomposition as

$$\tilde{G} = \tilde{C}_x(G_x + G_{xy}/2) + \tilde{C}_y(G_y + G_{xy}/2), \quad (6)$$

where $\tilde{C}_x$ and $\tilde{C}_y$ are measures of curvatures of the separable load-displacement curves.

Rigorous analytical formulations assume that the specimen is infinitely wide and the material deforms elastically. Neither is true in practical applications. Hence, the relevance of the analytical solutions to ductile specimens of finite width was investigated by finite element studies. In elastic case the solutions obtained from finite element method were the same as those for infinitely wide specimens [21]. In the nonlinear finite element study on center-cracked specimens, a generalized Ramberg-Osgood stress-strain relation was adopted to characterize the material nonlinearity [11,21]. When the applied stress σ, perpendicular to the crack is small

($\sigma/\sigma_y < 0.3$) the global energy rate G and the J-integral decrease with increasing biaxiality. However as σ is increased G and J exhibit a minimum near $k = 1$. The biaxial effects on stress intensity, strain intensity and plastic zone also are similar to that on J.

The finite element studies were expanded to analyze non-self-similar slow crack growth problems, incorporating incremental plasticity theory and kinematic hardening law in the computer code [29]. A fine mesh of four noded isoparametric elements was used to insure near-tip accuracy without employing a singularity element. The plastic energy was assumed to vary linearly with crack growth as in the self-similar crack growth case. The angle of crack extension was predicted using the maximum opening stress criterion. Several crack orientations and biaxial load ratios were studied. Critical loads were calculated by observing the load at which the plastic energy due to a virtual crack extension at constant load is larger than the stable growth plastic energy increment. At this point the crack can grow continually without further increase in applied load indicating the onset of unstable fracture. For the 2024-T3 aluminum alloy studied the direction of growth does not change significantly from the initial direction. For a more ductile material, however, an extrapolation of the predicted paths indicates that when large amounts of slow crack growth are involved, the deviation from the initial direction of growth may be significant. Qualitatively, the

results are consistent with observations on mixed mode fracture discussed in the literature.

The experimental studies have a great significance in biaxial loading situation because a rigorous elastic-plastic analysis of ductile fracture does not exist. Hence, experiments were performed on center-cracked specimens of metallic and non metallic materials including thick plate specimens of plexiglass and poly vinyl chloride (PVC) and thin sheets of aluminum alloys 2024-T3, 7075-T6 and 6061-T4. Cruciform specimens were used with gripping area divided into several tabs to prevent constraint to deformation of the gauge section. The crack was oriented parallel to one of the load axes. The brittle polymer, plexiglass exhibited considerable scatter in the data. A dominant effect of biaxiality was not observed on fracture load and fracture toughness. If there was a small effect it could not be detected due to the scatter in the data. In the more ductile PVC, the variation was smooth; the fracture load decreased steadily with increasing biaxiality, but the decrease was only about 10 percent. In plexiglass the crack orientation changed from the initial direction with lateral loading, and the deviation increased with increasing load biaxiality. On the other hand, in PVC the crack grew in a self-similar manner under all biaxialities. Among the aluminum alloys tested 7075-T6 is the least ductile and 6061-T4 in the most ductile. In 7075-T6 the remote fracture stress increased continually with increasing biaxiality, k from 0 to 1.8. On

the other hand in the more ductile 2024-T3 and 6061-T4 alloys, a maximum in remote fracture stress was observed around k = 1.0. One factor contributing to higher fracture stress at higher biaxiality is the stiffening effect of lateral load. In uniaxial testing stiffeners have been found to increase fracture stress by about 10 percent.

The influence of the biaxial loading on fracture observed in these studies do not conform to any of the analytical and finite element models developed. This is true of metals and polymers as well as ductile and brittle materials. The results indicate the need for an improved model to characterize biaxial load effects. One conclusion that can be drawn from these results is that he influence of load biaxiality in fracture is not significant when crack is perpendicular to one of the axes.

Note: Details of the study are furnished in Appendix A: A-3, A-6, A-7, A-9, A-10, A-12-A-15, A-21 and A-22.

V. BUCKLING OF SPHERICAL SHELLS

Buckling of spherical shells has important practical applications. Shell structures designed according to static analysis may fail under conditions of dynamic loading. Similarly other factors such as nonlinearities in material and design, imperfections and time-dependent deformation also affect buckling characteristics. Hence, some of these factors have been studied using a finite difference analysis [30-33].

In the first phase of this task, dynamic buckling of spherical shells with imperfections was studied using elastic solutions [30]. The inflection point of the load-displacement curve in the range where a sharp jump in peak average displacement occurs for a small change in load amplitude was taken as the buckling load. Step loading of infinite duration and right triangular pulse loading were used. The result demonstrates that static response is more sensitive to initial imperfections than dynamic response and static buckling load decreases more rapidly than dynamic buckling load. Pulse duration has a very significant effect on the magnitude of dynamic buckling load. Step loading produces a more severe response than a triangular pulse.

Subsequent studies were based on large strain elastic-plastic analysis. Static and dynamic buckling loads were obtained for axisymmetric spherical caps with imperfections [31, 32]. Both perfectly plastic and strain hardening behavior

were considered. Strain hardening was represented by kinematic hardening theory, so that Bauschinger effect was accounted for. Although plastic deformation has an important effect on small imperfections, as the size of the imperfection increases plastic deformation becomes less important in static loading. In dynamic loading the influence of both imperfection and plasticity increases with increasing thickness to radius ratio.

Additional investigations were carried out on the influence of imperfections and creep deformation on buckling of shallow spherical shells [32]. For nonlinear creeps both time-hardening and strain-hardening rules were employed. Results indicate that strain hardening yields better estimates of shelf life than time hardening. When compared with the experimental data from test specimens that possess very small departures from sphericity, it is observed that, in order to have a satisfactory prediction of both creep buckling times and deformations, the analysis should adopt a mathematical creep model which includes the primary, secondary and tertiary creep, in addition to taking into account the presence of imperfections.

Note: Details of the study are furnished in Appendix A: A-8, A-16-A-18.

VI. PLASTIC DEFORMATION IN THREE-DIMENSIONAL FRACTURE SPECIMENS

In spite of numerous attempts to develop a geometry-independent criterion for ductile fracture, the achievement of this goal has remained elusive. Without exception, all the suggested criteria show a marked thickness and geometry dependence limiting their predictive capabilities, making the application of these criteria to practical specimens difficult. The difficulty arises because of the inability of the methods to account for the variations in the nature of crack-tip plastic deformation with changes in geometry and size of the specimens. Hence, efforts have been made to study plastic deformation in three-dimensional specimens under load before crack growth initiation and during stable growth. A finite element program incorporating incremental plasticity constitutive relation, arbitrary hardening law and multilinear stress-strain representation has been developed. Details of the program are furnished in Appendix B. Some characteristics of the crack-tip plastic deformation has been studied using this program [34-36].

A comparative study of three-dimensional stress intensity factor solutions was made for through cracks in finite thickness isotropic elastic plates [34]. A wider range of thicknesses than previously studied were investigated. Both singular and non-singular elements were employed to investi-

gate thickness dependencies and variations. Grid characteristics yielding optimal solutions with minimal necessary degrees of freedom for convergence are discussed. The results indicate that accurate solutions can be obtained with all four techniques studied. The required degrees of freedom, however, are much greater for the grids employing only nonsingular elements. In all cases the singular element results predict a physically unrealistic kink in the stress intensity factor variation near the free surface. This variation, however, is not predicted by grids only employing nonsingular elements. Contrary to speculative reports of other authors, 20-node quarter-point elements predicted equally accurate results compared to 15-node "collapsed" quarter-point elements. For convergence, 20-node quarter-point element grids require fewer degrees of freedom than 15-node quarter-point grids.

The finite element program was used to examine the local elastic-plastic deformation response of three-dimensional cracked specimen [35]. The formulation employed incremental J_2 flow theory of plasticity with an arbitrary hardening response. The generality of the hardening law employed allowed for user determined hardening input. Infinitesimal displacements and strains were assumed. A through crack problem was solved as an example, employing three hardening assumptions (isotropic, kinesmatic and mixed). While the predicted plastic deformation in the region of the crack front predicted by these models are similar, several fundamental characteristics of each

assumption can be observed. The qualitative differences between hardening assumptions are consistent between the plasticity measures, allowing for direct comparison with experimental observation.

The three-dimensional elastic-plastic finite element program was employed to investigate the effect of specimen thickness on local crack front yielding characteristics in a cracked specimen [36]. Specimen thicknesses investigated range from well beyond ASTM plane-strain requirements to thin sheet dimensions. The yield zones calculated in this work demonstrate the transition from dilatational to distortional dominance ahead of the crack tip as a function of thickness (equivalent to a transition from plane strain to plane stress). The magnitude as well as the extent of yielding is shown to be highly thickness dependent. The results of this study also demonstrate that two-dimensional analysis based on plane strain (for thick specimens) or plane stress (for thin specimens) can fail to accurately model the local response when simple standards would dictate otherwise.

Work toward furthering the understanding of the slow growth process through the three-dimensional investigation is in progress. Both theoretical and experimental studies are being performed to delineate the parameters influencing the fracture process.

Note: Details of the study are furnished in Appendix A: A-28-A-30.

VII. CONCLUDING REMARKS

The research work presented in this report encompasses various aspects of fracture. The program included analytical, finite element and experimental studies. Considerable emphasis was given to nonlinear and biaxial effects in fracture of structural materials. As a result of these studies our understanding of fracture of materials in real life situations has improved considerably. However, due to the complex nature of the fracture problem, many aspects of fundamental importance are not well understood and additional studies are needed.

Presently further studies on plasticity in three-dimensional fracture specimens are being pursued. The plastic strain field are being obtained from finite element study for different thickness and are being compared with the experimental plastic zone measurement. The finite-element program will later be modified to include large strain model and changes during stable crack growth will be studied. These findings will be compared with experimental results. It is hoped that this research program will culminate in a truly predictive constitutive theory for fracture.

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